UNIVERSITY OF MANITOBA

DATE: April 22, 2008 FINAL EXAMINATION
PAPER # 534 TITLE PAGE
EXAMINATION: Engineering Mathematical Analysis 1 TIME: 3 hours
COURSE: MATH 2130 EXAMINER: G.I. Moghaddam

NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SEAT NUMBER: ______________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 10 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	6	
3	8	
4	8	
5	9	
6	12	
7	11	
8	10	
9	6	
10	6	
11	8	
12	8	
Total:	100	

[8] 1. Find the distance between the two lines $\dfrac{x+1}{2} = \dfrac{y-3}{3} = z+4$ and $x = 1-t, \quad y = -2t, \quad z = -3+2t.$

[6] 2. Find an equation for the tangent plane to the surface $x^3 + 3\,y^2 - 3\,z^2 = 3$ at the point $(3,\,1,\,3)$.

[8] 3. Let $u(x, y) = f(x^3 + y^2) + g(x^3 + y^2)$ such that f and g are differentiable functions. Show that

$$2y\,\frac{\partial u}{\partial x} - 3x^2\,\frac{\partial u}{\partial y} = 0.$$

[8] 4. Given that the equations

$$e^x + \sin y = u^2 - v^2, \quad \text{and} \quad e^y + \sin x + 2u^2 + v^2 = 0$$

define u and v as functions of x and y find $\dfrac{\partial u}{\partial x}$. Simplify your answer.

[9] 5. Evaluate the following double integral.

$$\int_0^1 \int_0^{\frac{1}{2}(1-y)} e^{x-x^2}\, dx\, dy$$

[12] 6. Find the absolute maximum and the absolute minimum of the function

$$f(x,y) = x^2 + 2xy - y^2$$

on the region bounded by $x = \sqrt{1-y^2}$, $y = x$ and $y = 0$.

[11] 7. Consider a thin plate with mass per unit area $\rho(x, y) = x^2 + y$ such that the edges of the plate are defined by the parabola $y = (x - 2)^2$ and the line $y = x$. **Set up but do not evaluate** double integrals for each of the following.

(a) First moment of the plate about the $y-$axis.

(b) Moment of inertia of the plate about the line $4x - 3y + 1 = 0$.

(c) Centre of mass of the plate.

[10] 8. Consider the double integral

$$\iint_R \sqrt{1 + \left[\frac{\partial}{\partial x}(y^2 - x^2)\right]^2 + \left[\frac{\partial}{\partial y}(y^2 - x^2)\right]^2}\ dA$$

where R is the region between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(a) Simplify the integral.

(b) Give a (natural) geometrical interpretation of the integral.

(c) Rewrite the integral in terms of polar coordinates and then evaluate the integral.

[6] 9. **Set up but do not evaluate** a **set** of iterated integrals to evaluate

$$\iiint_V dV$$

where V is a region in $\mathbf{R}^3$ bounded by the planes

$$z = 0, \quad z = 3x, \quad x + z = 4, \quad y = 0, \quad y = 2.$$

[6] 10. (a) Find the spherical coordinates of the point P with cartesian coordinates $(\sqrt{2},\, \sqrt{2},\, 2\sqrt{3})$.

(b) Find the cylindrical coordinates of the point Q with spherical coordinates $(2\sqrt{2},\, \frac{\pi}{6},\, \frac{\pi}{4})$.

[8] 11. Find the volume of the region inside the cylinder $x^2 + y^2 = 1$ and between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$.
(Hint: you may use cylindrical coordinates system.)

[8] 12. A solid half ball (semisphere) V of radius 3 has density ρ depending on the distance $\mathfrak{R}$ from the centre of the base disk. The density is given by $\rho = k(6-\mathfrak{R})$ where k is a constant. Use **spherical coordinates** system to find the **mass** of the half ball.